Part I

CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called a circular motion with respect to that fixed (or moving) point.

ANGULAR VELOCTIY (ω)

Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} \; ; \quad \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

Instantaneous Angular Velocity

The rate at which the position vector of a particle with respect to the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relative Angular Velocity

$$\omega_{AB} = \frac{(V_{AB})_{\perp}}{r_{AB}}$$

here $V_{AB_{\perp}}$ = Relative velocity perpendicular to position vector AB

Relation between speed and angular Velocity : $v = r\omega$ is a scalar quantity ($\vec{\omega} \neq \frac{\vec{v}}{r}$)

ANGULAR ACCELERATION (α)

Average Angular Acceleration

Let ω_1 and ω_2 be the instantaneous angular speed at time t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_1 - \omega_2}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

It is the limit of average angular acceleration as Δt approaches zero, that is

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

Part II

RADIAL AND TANGENTIAL ACCELERATION

$a_t = \frac{dv}{dt}$ = rate of change of speed and $a_r = \omega^2 r = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$

Angular and Tangential Acceleration Relation

$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$ or $a_t = r\alpha$

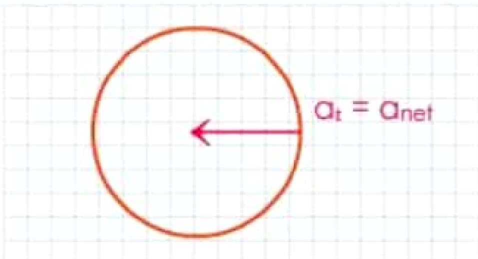
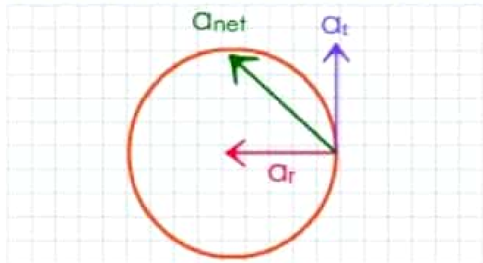
RELATIONS AMONG ANGULAR VARIABLES

Equations of Rotational Motion

$$\omega = \omega_0 + \alpha t$$

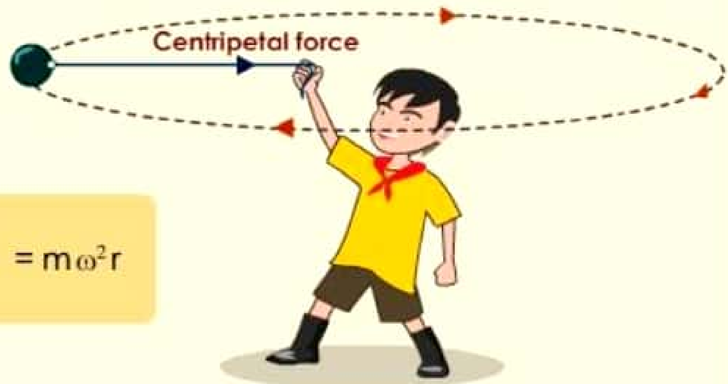
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

Uniform Circular Motion	Non-Uniform Circular Motion
Speed of the particle is constant i.e., ω = constant	Speed of the particle is not constant i.e., $\omega \neq$ constant
$a_t = \frac{d\lvert\vec{v}\rvert}{dt} = 0$; $a_r = \frac{v^2}{r} \neq 0$ $\therefore$ $a_{net} = a_r$	$a_t = \frac{d\lvert\vec{v}\rvert}{dt} \neq 0$; $a_r \neq 0$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
	

CENTRIPETAL FORCE

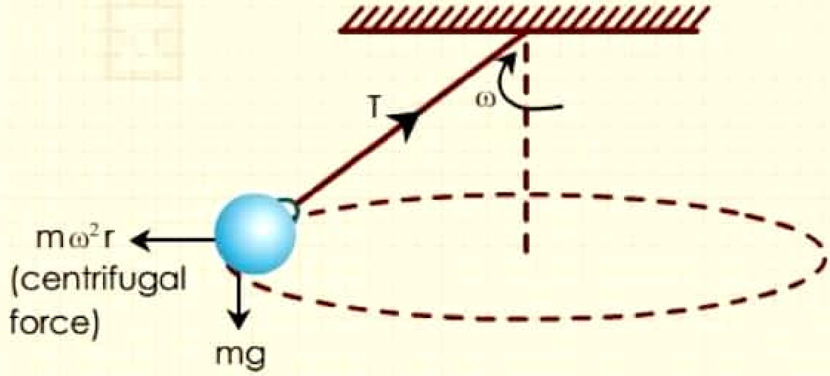
Centripetal force is the necessary resultant force towards the centre.



$$F = \frac{mv^2}{r} = m\omega^2 r$$

Part III

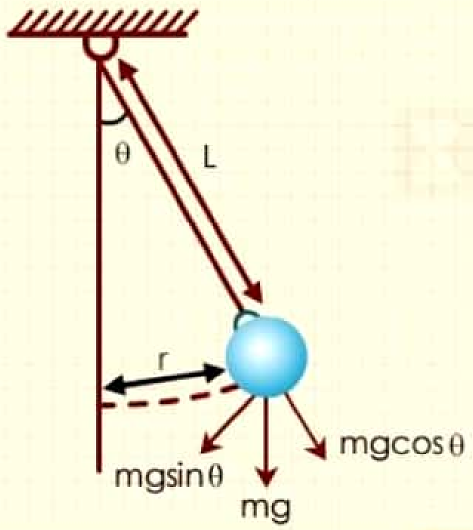
CENTRIFUGAL FORCE



Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion (in that frame)

$$F_c = m\omega^2 r$$

SIMPLE PENDULUM



Balancing Horizontal Forces:

$$T \sin\theta = m\omega^2 r$$

Balancing Vertical Forces:

$$T - mg\cos\theta = mv^2/L \implies T = m(g\cos\theta + v^2/L)$$

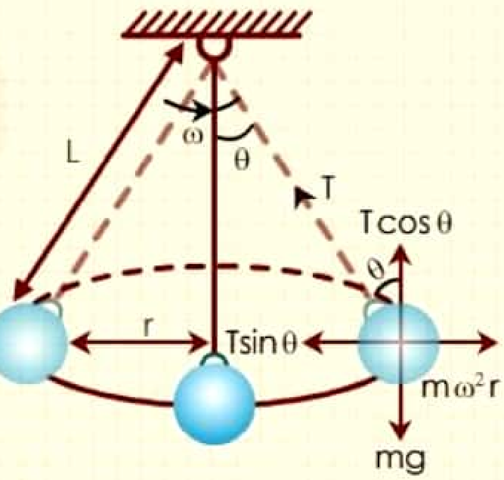
$$|\vec{F}_{net}| = \sqrt{(mg\sin\theta)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\theta + \frac{v^4}{L^2}}$$

CONICAL PENDULUM

FBD of ball shows:

$T\sin\theta = m\omega^2 r$ = centripetal force

$T\cos\theta = mg$



FBD of ball w.r.t ground

speed $v = \dfrac{r\sqrt{g}}{(L^2 - r^2)^{1/4}}$ and Tension $T = \dfrac{mgL}{(L^2 - r^2)^{1/2}}$

CIRCULAR TURNING ON ROADS

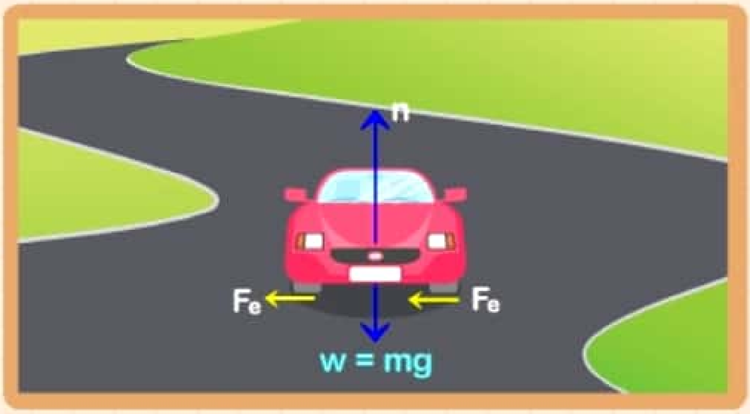
Centripital force required for turning is provided in following ways.

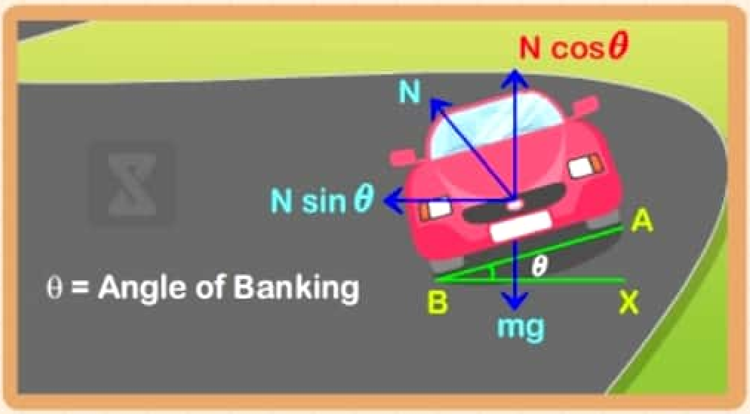
BY FRICTION ONLY

For a safe turn without sliding:

Safe Speed $v \leq \sqrt{\mu r g}$

- The safe speed of the vehicle should be less than $\sqrt{\mu r g}$
- The coefficient of friction should be more than v^2/rg.





BY BANKING OF ROADS ONLY

From FBD of car:

$$N\sin\theta = \frac{mv^2}{r} \quad \& \quad N\cos\theta = mg$$

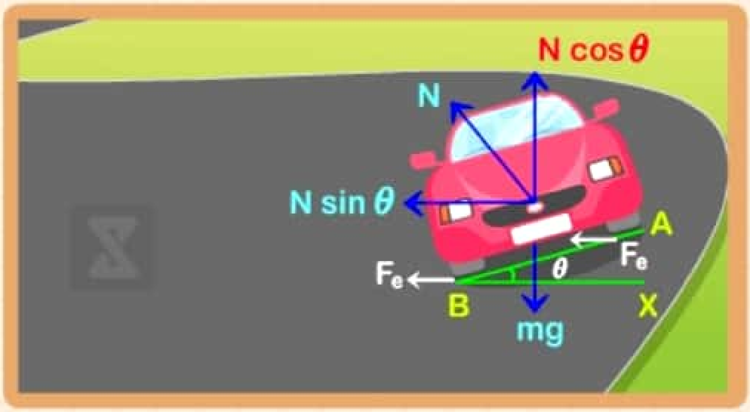
From these two equations, we get

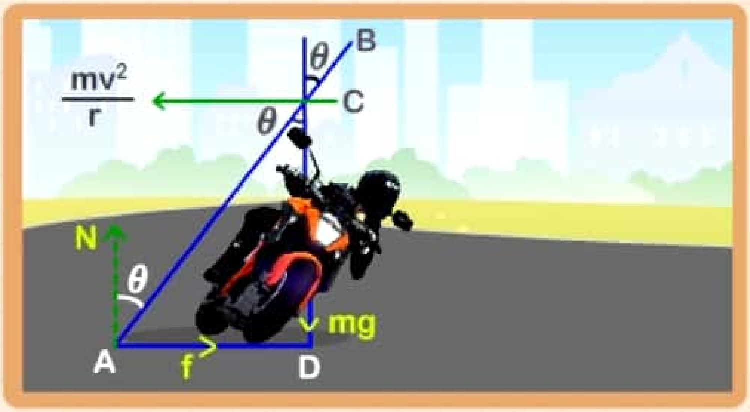
$$\tan\theta = \frac{v^2}{rg} \quad \& \quad v = \sqrt{rg\tan\theta}$$

BOTH FRICTION AND BANKING OF ROADS

Maximum safe speed $v_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$

Minimum safe speed $v_{max} = \sqrt{\frac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}$



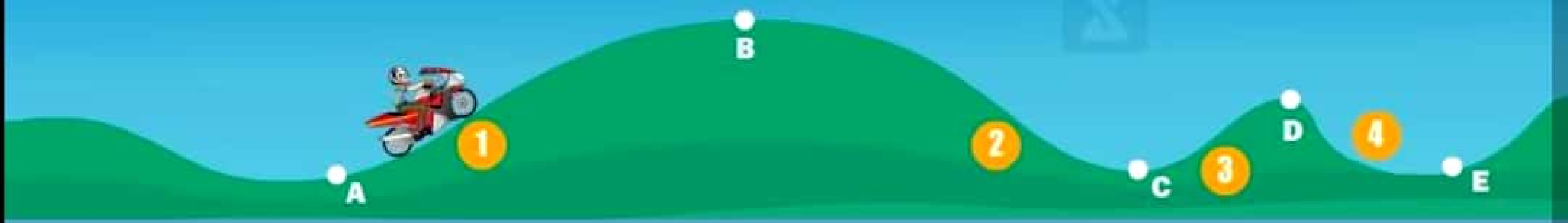


BIKE ON A CIRCULAR PATH

$$\frac{AD}{CD} = \frac{v^2}{rg} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

Thus, the cyclist bends at an angle $\tan^{-1}[v^2/rg]$ with the vertical.

MOTORCYCLIST ON A CURVED PATH



A cyclist having mass m moving with constant speed v on a curved path

We divide the motion of the cyclist in four parts :

1 From A to B	2 From B to C	3 From C to D	4 From D to E

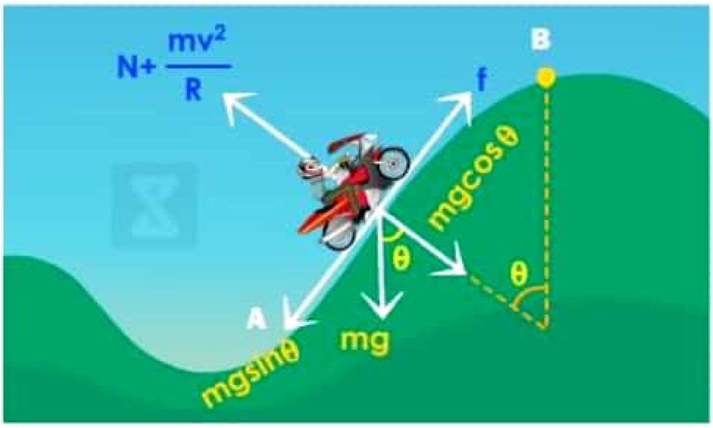
MOTION OF CYCLIST FROM A TO B

(1 and 3 are same type of motion)

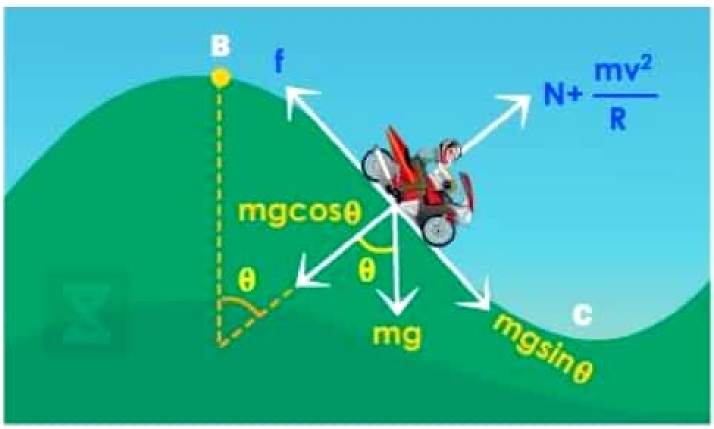
$$N + \frac{mv^2}{R} = mg\cos\theta \quad : \quad f = mg\sin\theta$$

AS CYCLIST MOVE UPWARD

In 1 and 3 normal force increases but frictional force decreases because θ decreases.



MOTION OF CYCLIST FROM B TO C



$$N + \frac{mv^2}{R} = mg\cos\theta \Longrightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

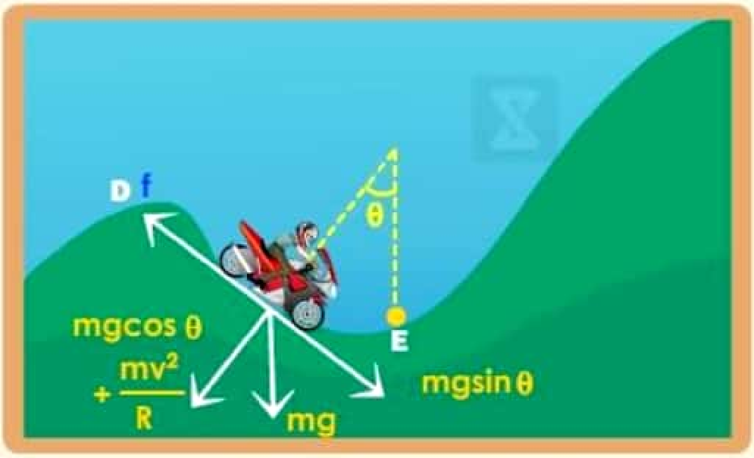
$$f = mg\sin\theta$$

From B to C, Normal force decreases but friction force increases because θ increases.

MOTION OF CYCLIST FROM D TO E

$$N = \frac{mv^2}{R} + mg\cos\theta \quad : \quad f = mg\sin\theta$$

From D to E, 'θ' decreases therefore mg cosθ increases whereas Normal force increases but frictional force decreases.



WORK, POWER, ENERGY

WORK

$W = \vec{F}.\vec{ds} = Fs\cos\theta$

$W = \tau\theta$

F = Force Applied

$\vec{ds}$ = Displacement

θ = Angle Between Force and Displacement

τ = Torque

θ = Angle of Rotation

POWER

$P = \frac{dW}{dt} = \frac{\vec{F}.\vec{ds}}{dt} = \vec{F}.\vec{v}$

ENERGY

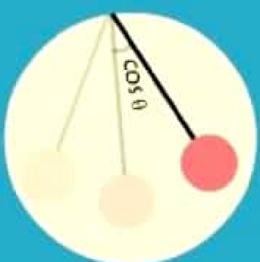
KINETIC ENERGY

$K.E._{Trans} = \frac{1}{2}mv^2$

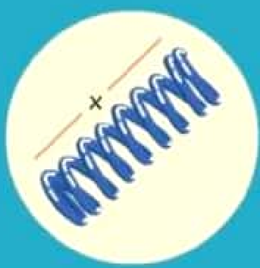
$K.E._{Rot} = \frac{1}{2}I\omega^2$

$K.E._{Rolling} = mv^2 + \frac{1}{2}I\omega^2$

POTENTIAL ENERGY



$PE_{Pendulum} = mgl(1-\cos\theta)$

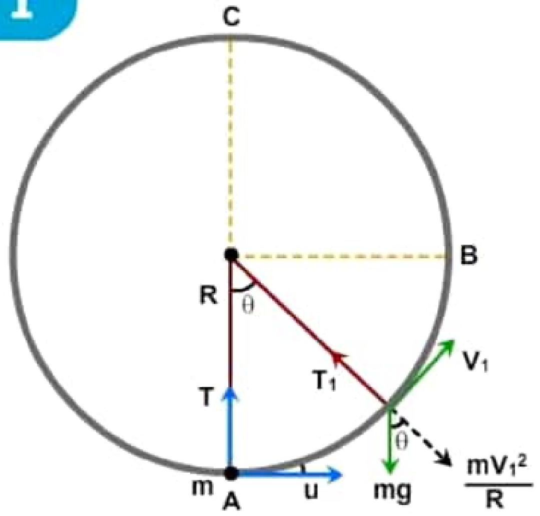


$PE_{spring} = \frac{1}{2}Kx^2$

$PE_{grav} = mgh$

VERTICAL CIRCULAR MOTION

1

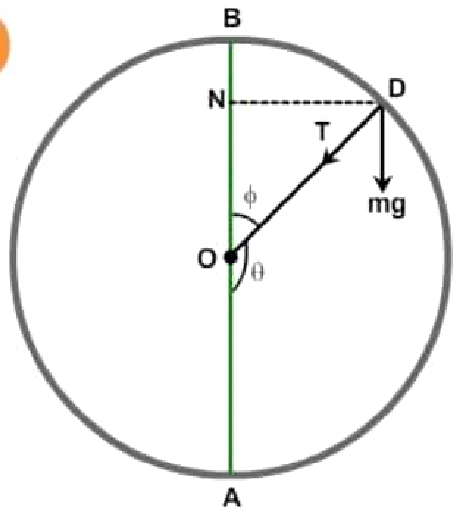


Ball will complete the circle

Condition: Initial velocity, $u > \sqrt{5gR}$

- Tension at A : $T_A = 6mg$
- Tension at B : $T_B = 3mg$
- If $u = \sqrt{5gR}$ ball will just complete the circle and velocity at topmost point is $v = \sqrt{gR}$

2

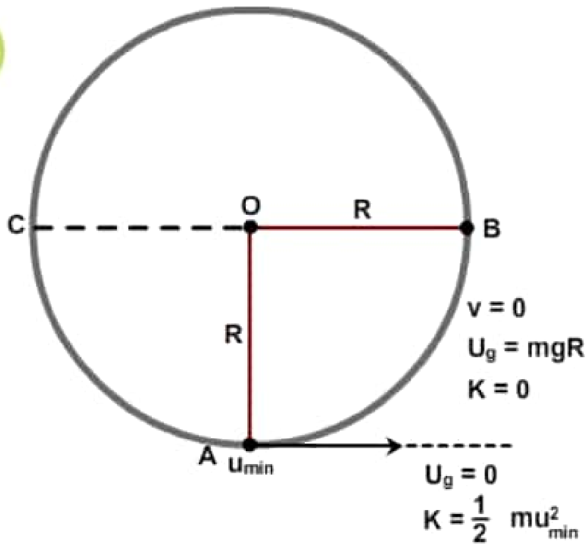


Ball will slack in between

Condition: $\sqrt{2gR} < u < \sqrt{5gR}$

- $\cos\phi = \dfrac{u^2 - 2gR}{3gR}$. ν

3



Ball will reach B

Condition: $u \leq \sqrt{2gR}$

- Ball will oscillate between CAB
- Velocity $v = 0$ but $T \neq 0$

Note: At height **h** from bottom of ball velocity will be, $v = \sqrt{u^2 - 2gh}$