REST AND MOTION

DISTANCE

- The length of the actual path traversed by the particle is termed as its distance.
- Distance = S = length of path ACB.
- Scalar quantity and is measured in meter. It can never decrease with time.

DISPLACEMENT

- The change in position vector of the particle for a given time interval is known as its displacement.
- Displacement = B – A
- It can decrease with time. Vector quantity and is measured in meter.

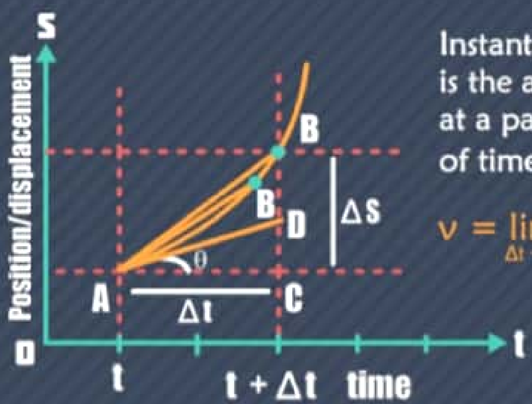
AVERAGE VELOCITY

$$\text{Average Velocity } (\vec{v}_{av}) = \frac{\text{Total Displacement}}{\text{Total Time Taken}} = \frac{\vec{B} - \vec{A}}{t}$$

AVERAGE SPEED

$$\text{Average Speed}(v_{av}) = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}} = \frac{S}{t}$$

INSTANTANEOUS VELOCITY



Instantaneous velocity is the average velocity at a particular instant of time.

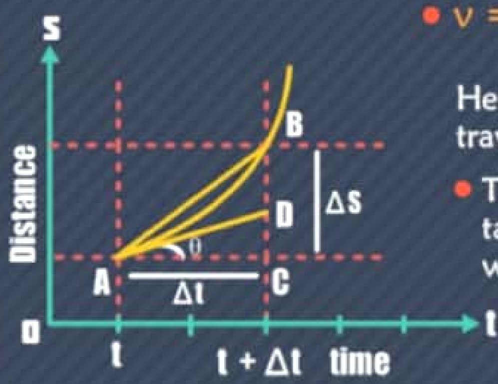
$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

INSTANTANEOUS SPEED

- The instantaneous speed is the speed at a particular instant of time.
- $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Here Δs is the distance travelled in time Δt.

- The slope of the tangent equal ds/dt, which is equal to the instantaneous speed at 't'.



$$v = \tan(\theta) = \frac{DC}{AC} = \frac{ds}{dt}$$

EQUATIONS OF MOTION

1. $v = u + at$
2. $v^2 - u^2 = 2as$
3. $s = ut + \frac{1}{2}at^2$
4. $S_{nth} = u + \frac{a}{2}(2n - 1)$

ACCELERATION

When the velocity of a moving object/particle changes with time, we can say that it is accelerated.

Average Acceleration

$$a_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

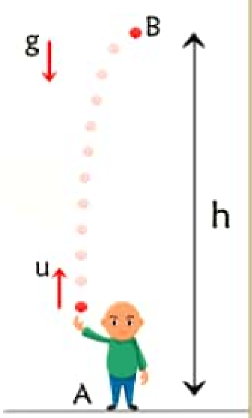
$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{av} = \frac{d\vec{v}}{dt}$$

REACTION TIME

It's the difference between the time when one see a situation to the time when one acts.

Reaction Time $\Delta t = t_1 - t_0$

MOTION UNDER GRAVITY

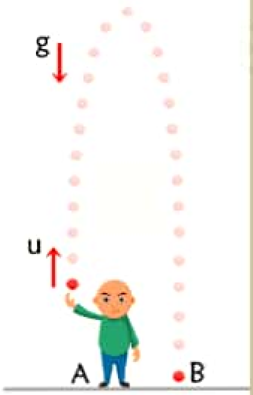


Sign Conventions

$u = +ve$
$h = +ve$
$v = 0$
$a = -g$

Equation of motion

$h = ut - 1/2gt^2$
$0 = u - gt$
$0^2 = u^2 - 2gh$

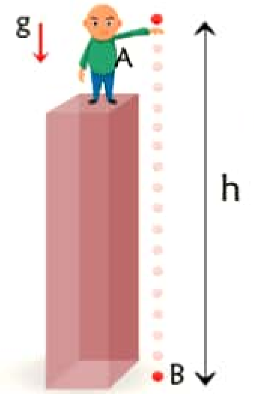


Sign Conventions

$u = +ve$
$h = 0$
$v = -ve$
$a = -g$

Equation of motion

$0 = ut - 1/2gt^2$
$-v = u - gt$
$v^2 = u^2 - 2g(0)$

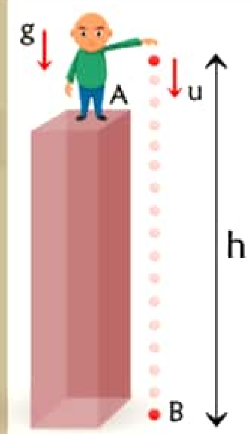


Sign Conventions

$u = 0$
$h = -ve$
$v = -ve$
$a = -g$

Equation of motion

$-h = 0(t) - 1/2gt^2$
$-v = 0 - gt$
$v^2 = (0)^2 + 2gh$
$v = \pm\sqrt{2gh}$

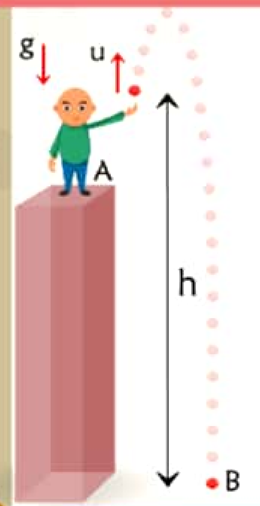


Sign Conventions

$u = -ve$
$v = -ve$
$a = -g$
$h = -ve$

Equation of motion

$-h = -ut - 1/2gt^2$
$-v = -u - gt$
$v^2 = u^2 + 2gh$



Sign Conventions

$u = +ve$
$v = -ve$
$a = -g$
$h = -ve$

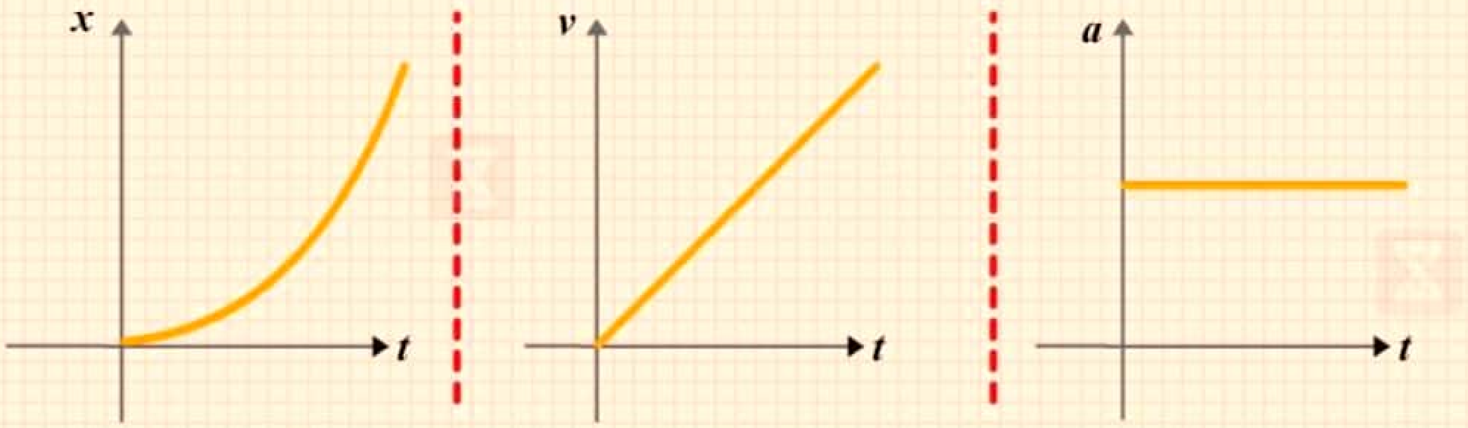
Equation of motion

$-h = ut - 1/2gt^2$
$-v = u - gt$
$v^2 = u^2 + 2gh$

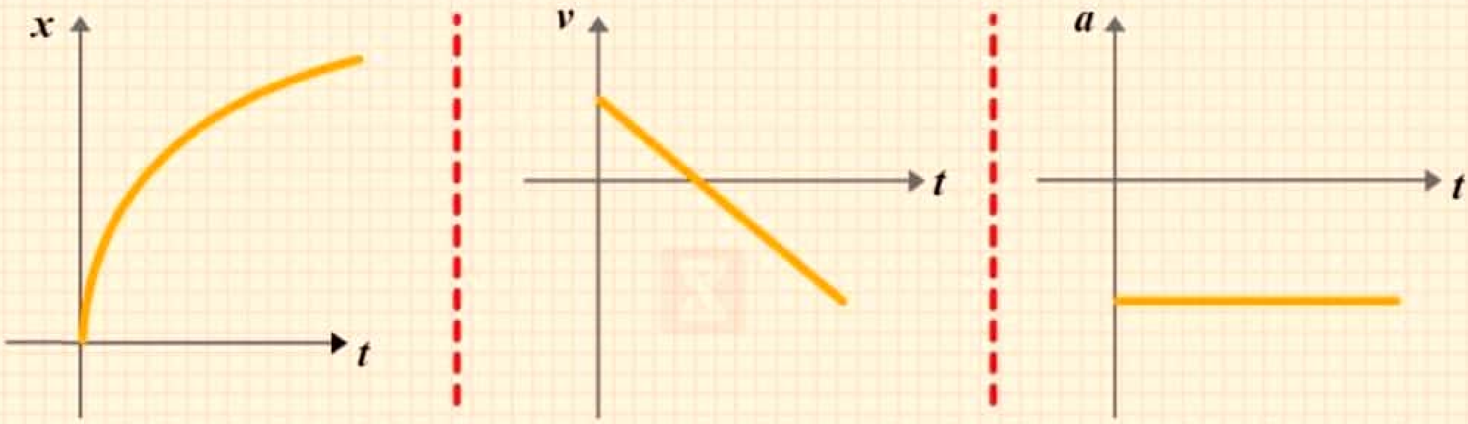
RECTILINEAR MOTION CASES

Distance | Velocity | Acceleration

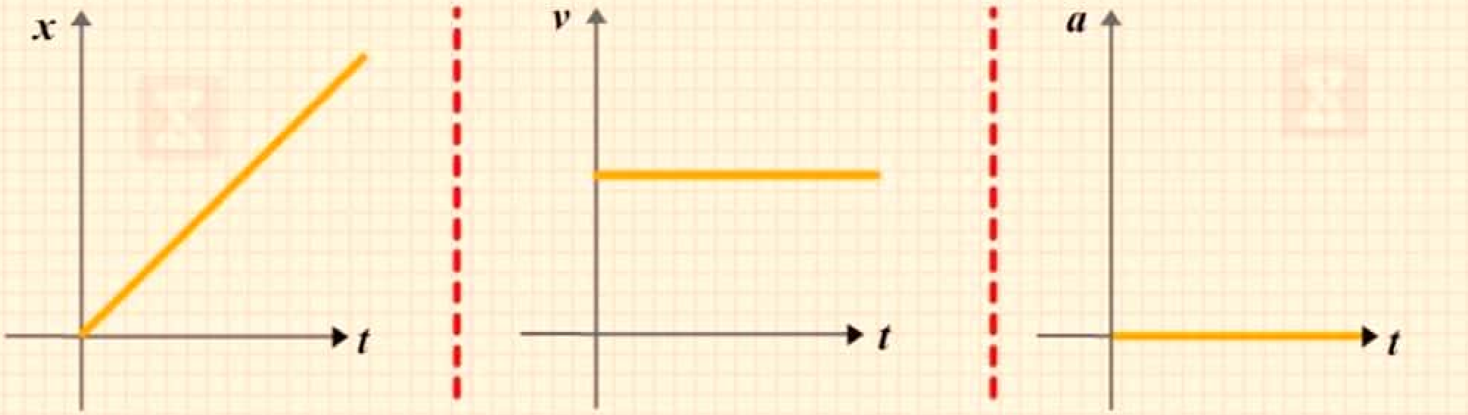
BODY MOVING WITH INCREASING VELOCITY

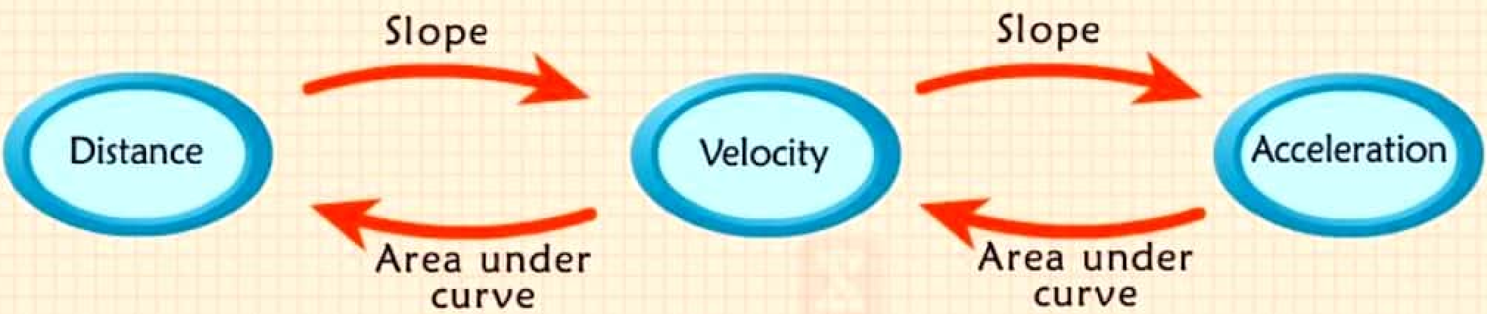


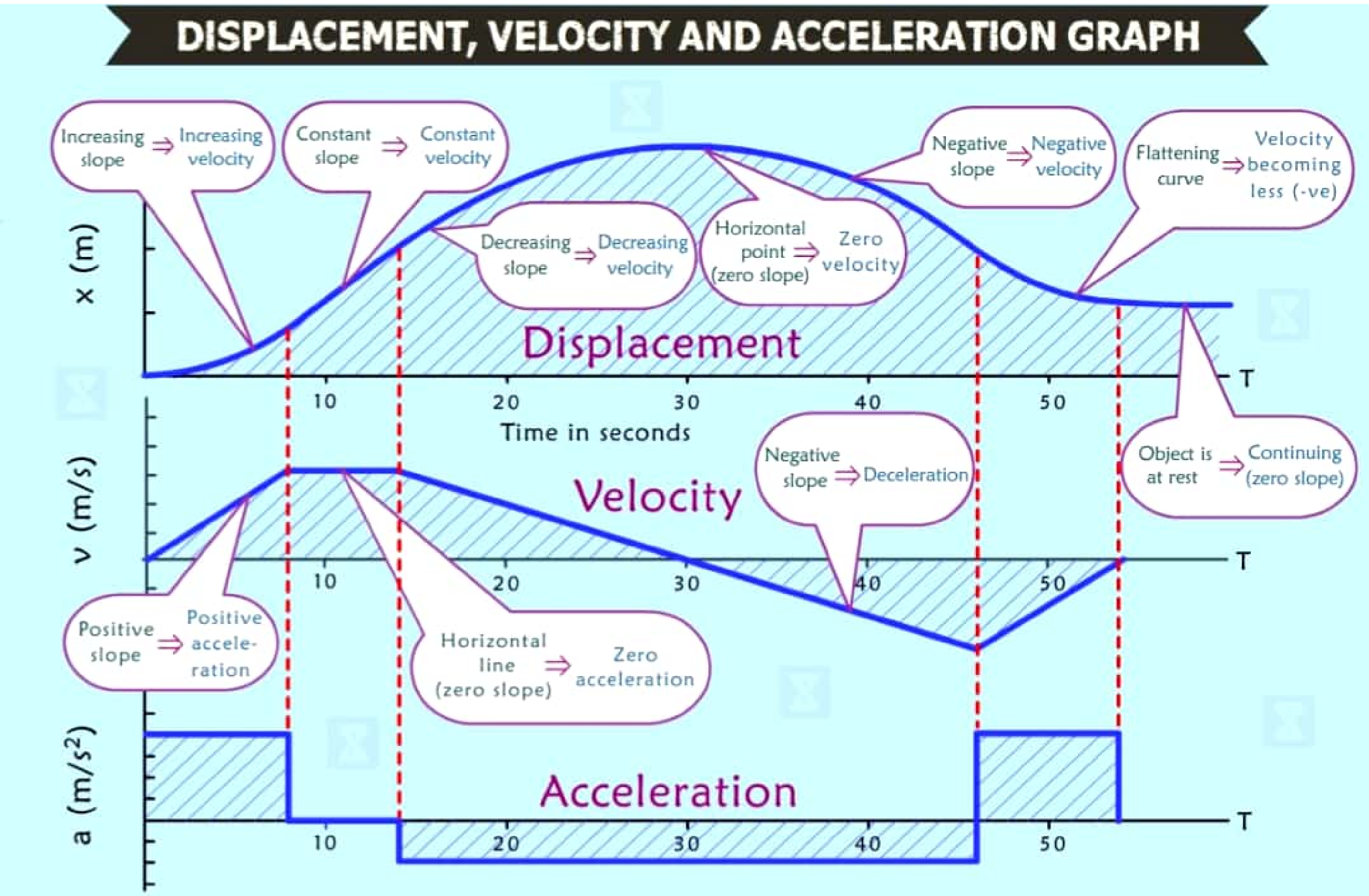
BODY MOVING WITH DECREASING VELOCITY



BODY MOVING WITH UNIFORM VELOCITY






DISPLACEMENT, VELOCITY AND ACCELERATION GRAPH
Increasing slope ⇒ Increasing velocity
Constant slope ⇒ Constant velocity
Decreasing slope ⇒ Decreasing velocity
Horizontal point (zero slope) ⇒ Zero velocity
Negative slope ⇒ Negative velocity
Flattening curve ⇒ Velocity becoming less (-ve)
x (m)
Displacement
10 20 30 40 50 T
Time in seconds
Object is at rest ⇒ Continuing (zero slope)
v (m/s)
Velocity
Negative slope ⇒ Deceleration
Positive slope ⇒ Positive acceleration
Horizontal line (zero slope) ⇒ Zero acceleration
a (m/s²)
Acceleration

RELATIVE VELOCITY

Relative velocity of A wrt B $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

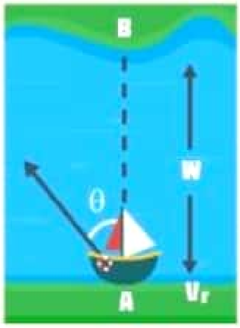
Relative acceleration of A wrt B $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

RIVER-BOAT PROBLEM

$\vec{V}_r$ = absolute velocity of river

$\vec{V}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

$\vec{V}_b$ = absolute velocity of boatman.



$$\vec{V}_b = \vec{V}_{br} + \vec{V}_r$$

Time taken by boatman to cross the river:

$$t = \frac{w}{V_{br}\cos\theta}$$

Displacement along x-axis when he reaches on the other bank:

$$x = (V_r - V_{br}\sin\theta)\frac{w}{V_{br}\cos\theta}$$

1. Condition when the boatman crosses the river in shortest interval of time-

$$t_{min} = \frac{w}{V_{br}}$$

2. Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started

$$\theta = \sin^{-1}\left(\frac{V_r}{V_{br}}\right)$$

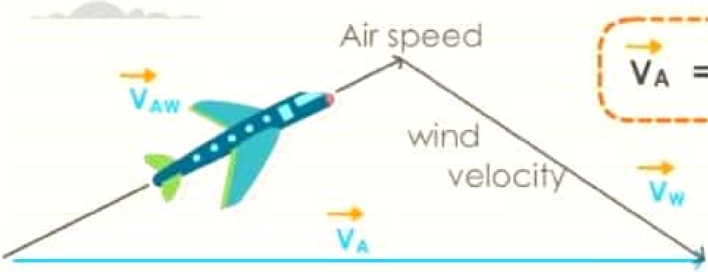
3. Shortest Path

when $V_r < V_{br} \rightarrow S_{min} = w$

when $V_r > V_{br} \rightarrow$

$$S_{min} = w\left(\frac{V_r}{V_{br}}\right)$$

AIRCRAFT WIND PROBLEM



$$\vec{V}_A = \vec{V}_{AW} + \vec{V}_W$$

$\vec{V}_{AW}$ = Velocity of aircraft wrt wind

$\vec{V}_W$ = Velocity of wind

$\vec{V}_A$ = Absolute Velocity of aircraft

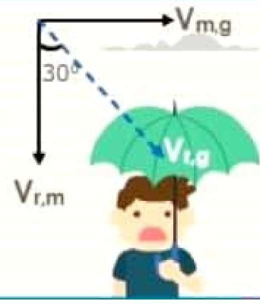
RAIN PROBLEM

$\vec{V}_{r,g}$ = Velocity of river wrt ground

$\vec{V}_{r,m}$ = Velocity of river wrt man

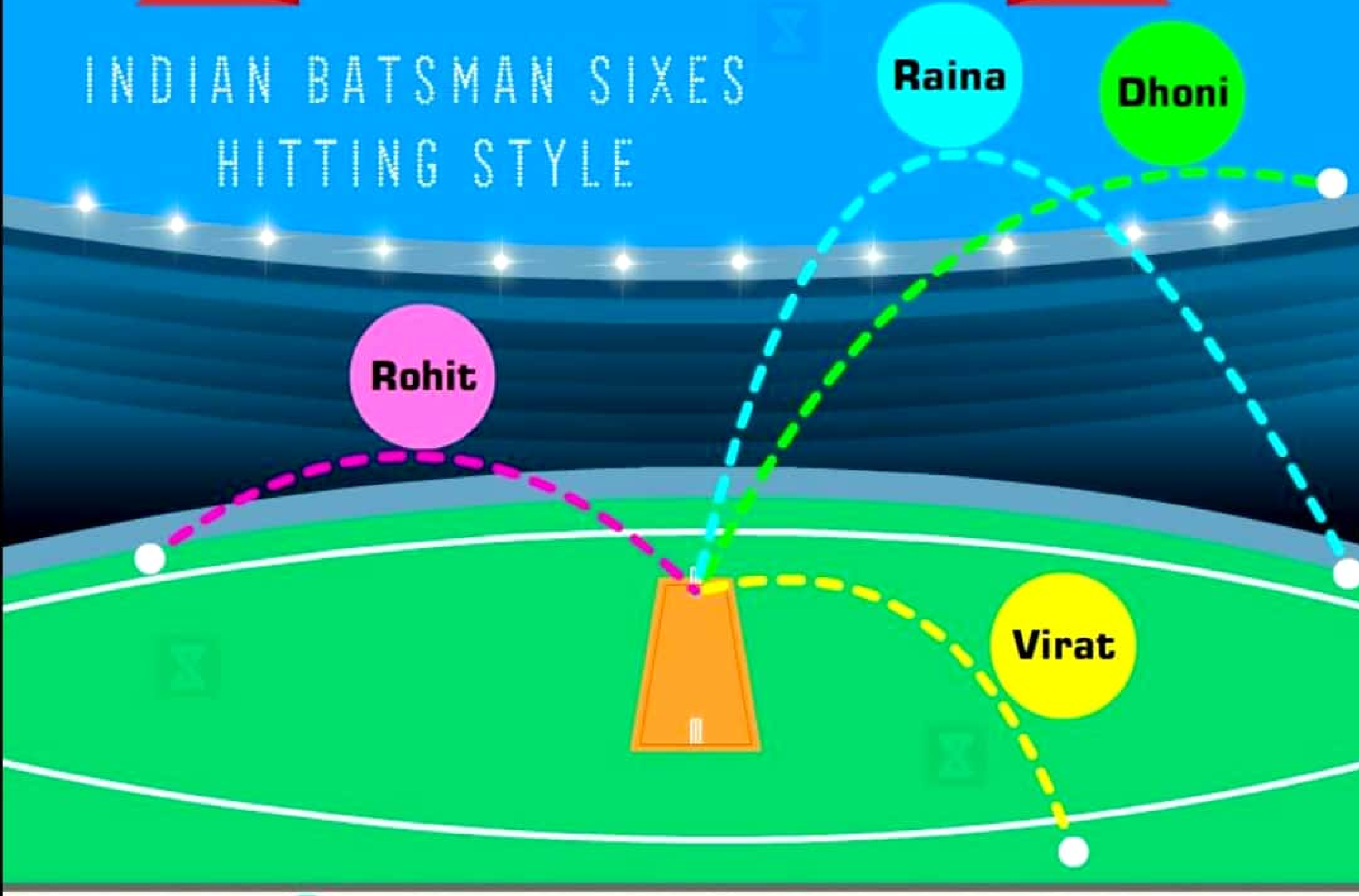
$\vec{V}_{m,g}$ = Velocity of man wrt ground

$$\vec{V}_{r,g} = \vec{V}_{r,m} + \vec{V}_{m,g}$$

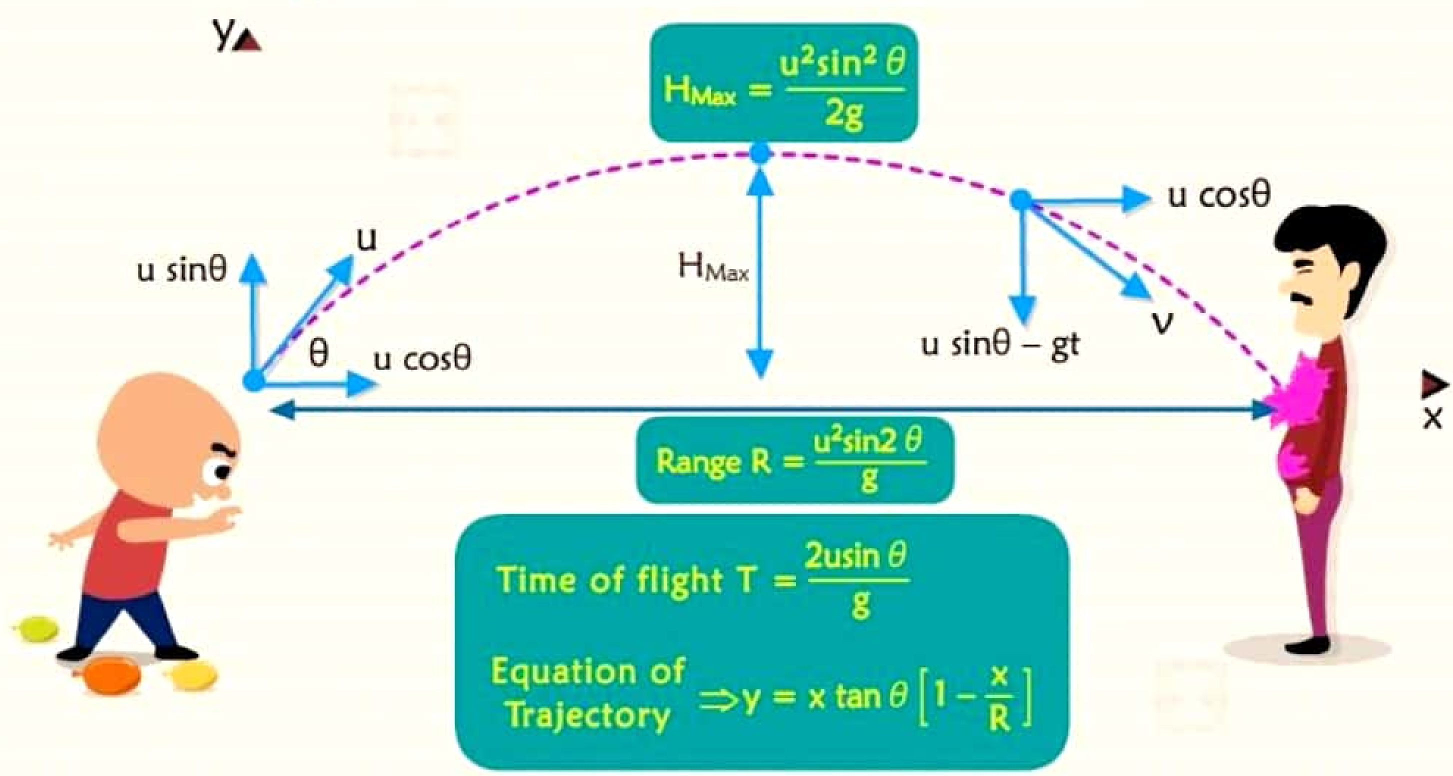


PROJECTILE MOTION

Part I

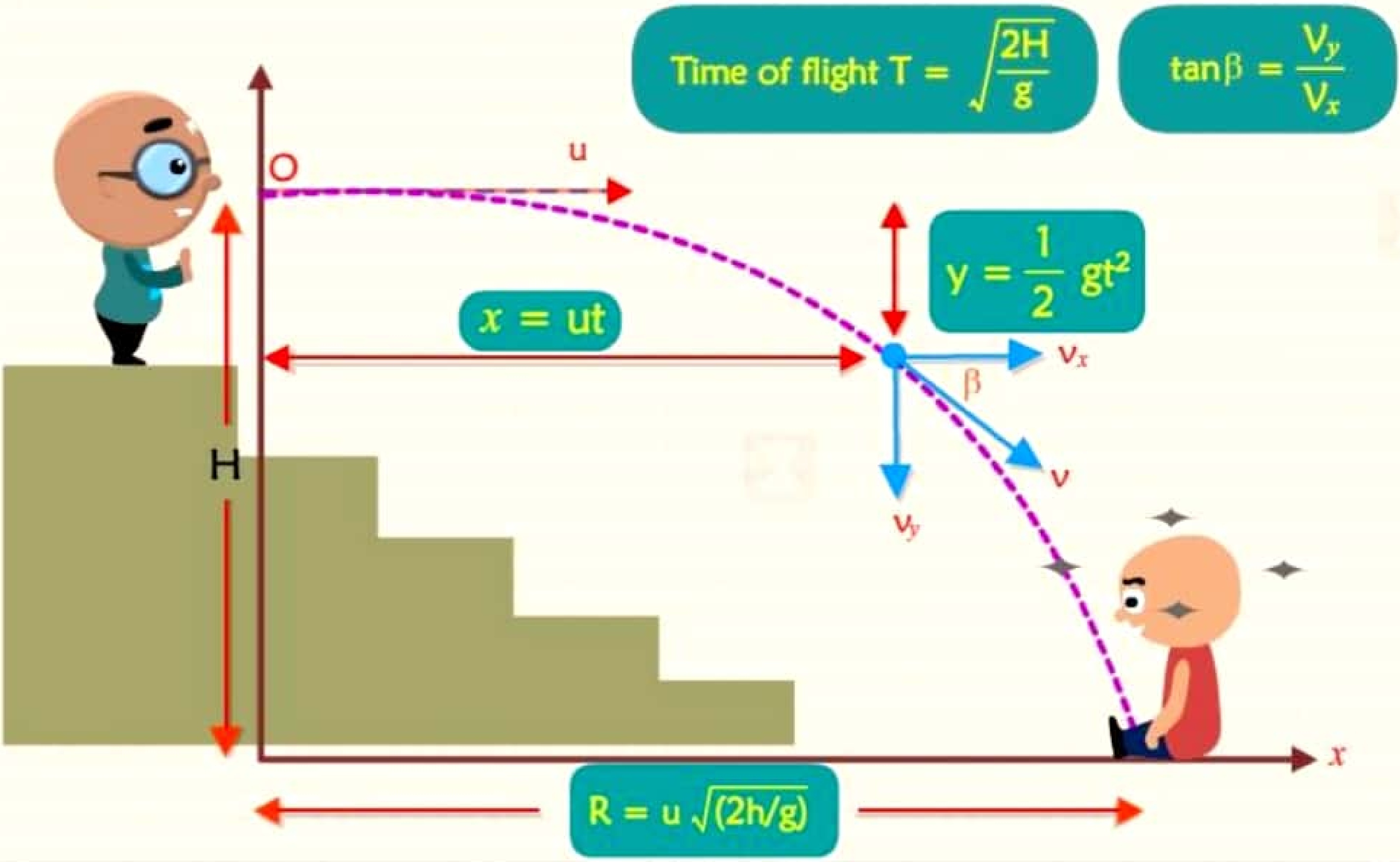


1 BASIC PROJECTILE MOTION

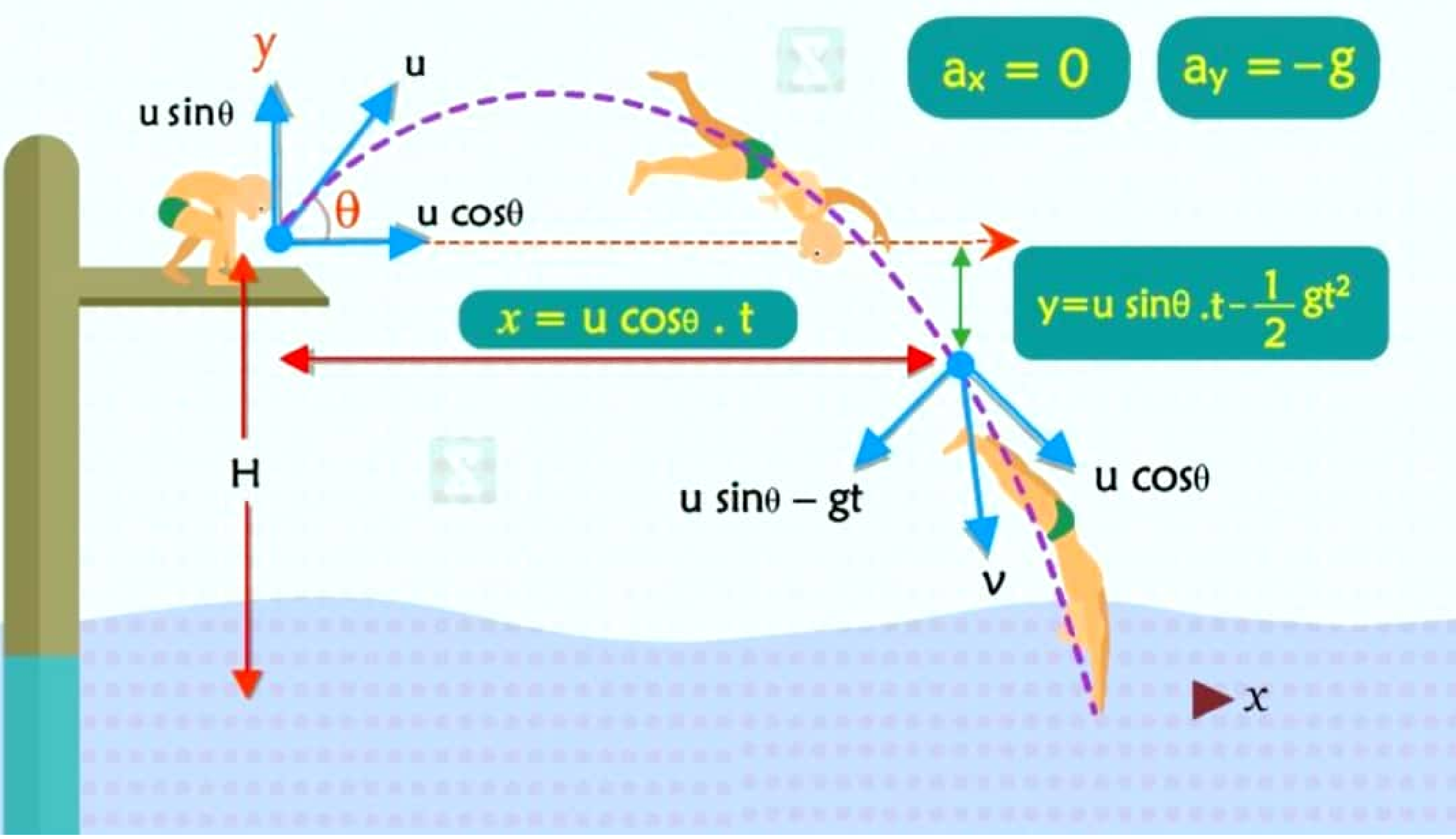


Part II

2 PROJECTILE FIRED PARALLEL TO HORIZONTAL

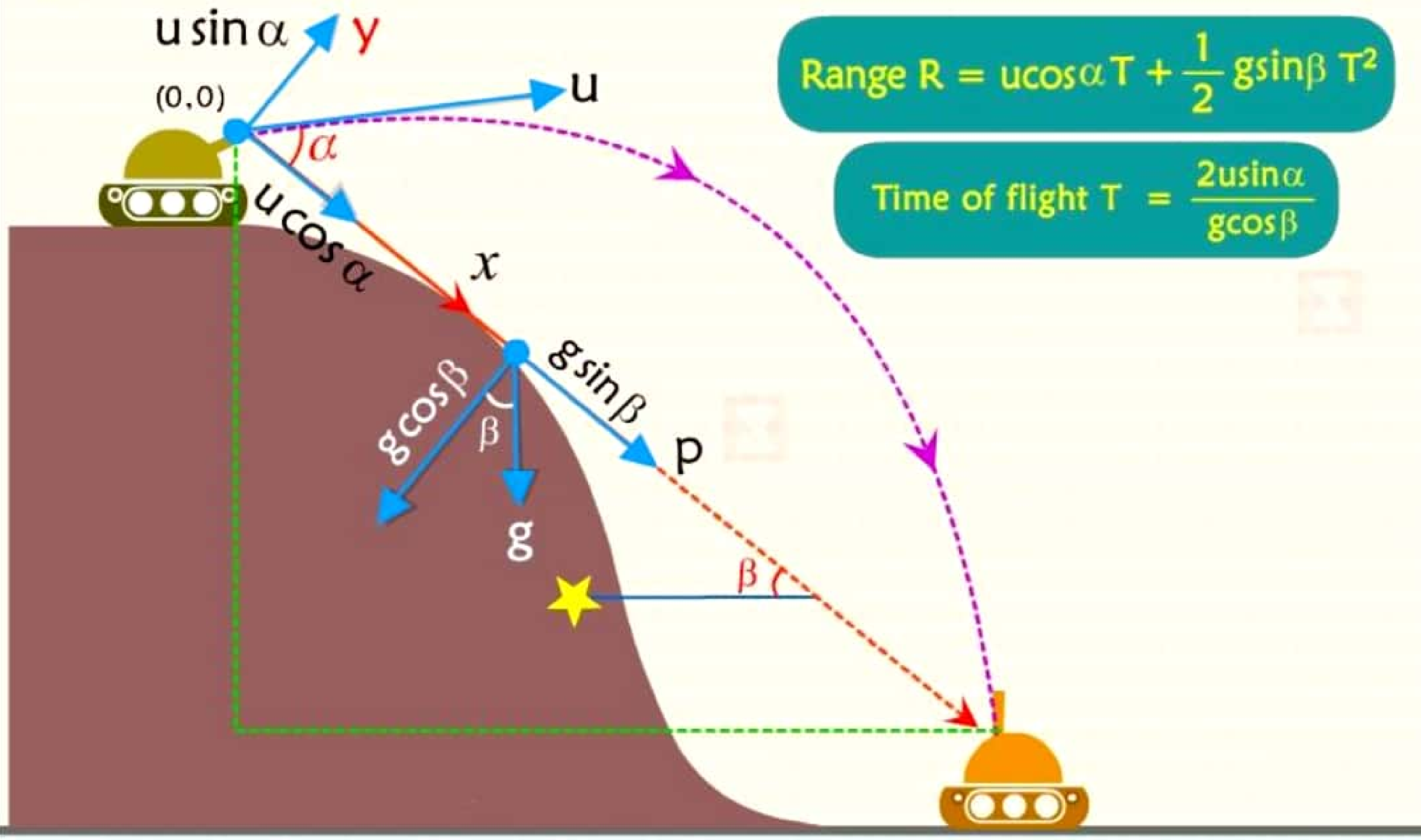


3 PROJECTILE AT AN ANGLE θ FROM HEIGHT 'H'



Part III

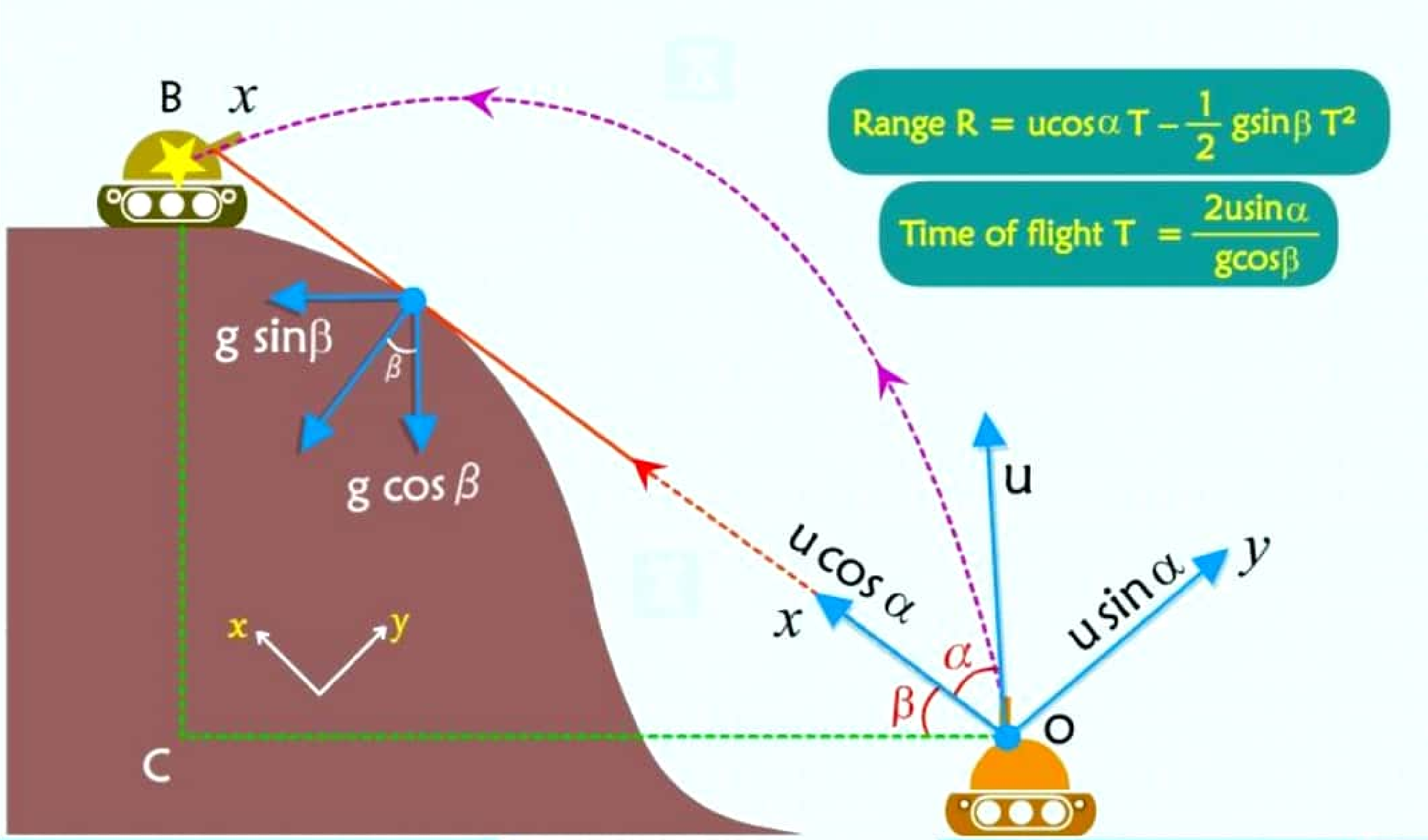
4 PROJECTILE MOTION DOWN THE INCLINED PLANE



$$\text{Range } R = u\cos\alpha\, T + \frac{1}{2} g\sin\beta\, T^2$$

$$\text{Time of flight } T = \frac{2u\sin\alpha}{g\cos\beta}$$

5 PROJECTILE MOTION UP THE INCLINED PLANE



$$\text{Range } R = u\cos\alpha\, T - \frac{1}{2} g\sin\beta\, T^2$$

$$\text{Time of flight } T = \frac{2u\sin\alpha}{g\cos\beta}$$