

# GRAVITATION



Do you know I attract you with a force ?



Yes, but it's too weak to be felt.



Force of attraction between them is gravitation and it is given by:

$$F = G \frac{Mm}{r^2} \quad G = \text{Gravitational Constant}$$

$$K.E = \frac{1}{2} mv^2$$

$$P.E = \frac{-GM_em}{r}$$

- $m$  = Mass of Satellite
- $r$  = Radius of Orbit

$$V_{esc} = \frac{GM_e}{R + h}$$

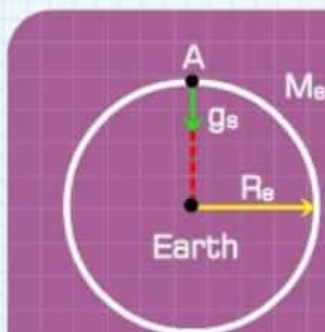
- $M$  = Mass of Earth
- $R$  = Radius of Earth
- $h$  = Height from Earth Surface



# GRAVITATIONAL FORCE

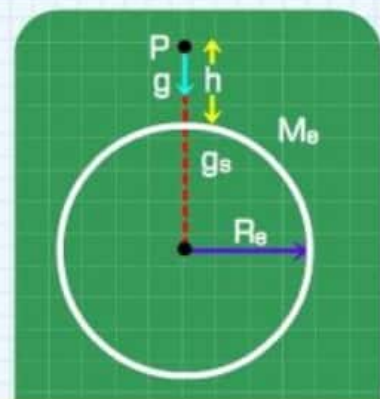
## Acceleration Due to Gravity

On the surface of earth



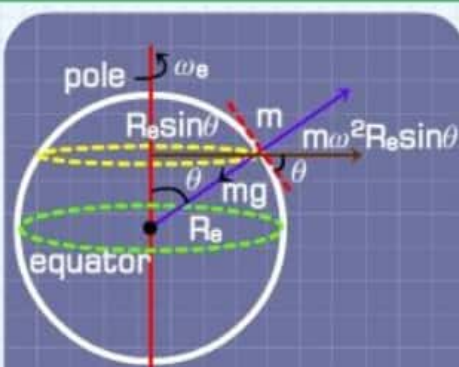
$$g = \frac{GM}{R^2} = 9.81 \text{ms}^{-2}$$

At height  $h$  from the surface of earth



$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = g \left(1 - \frac{2h}{R}\right) \text{ if } h \ll R$$

Effect of rotation of earth at latitude

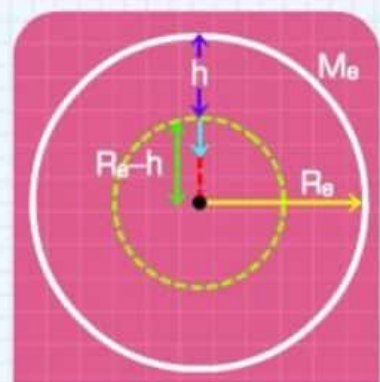


$$g' = g - R\omega^2 \sin^2 \phi$$

At equator,  $\phi = 90^\circ$ ,  $g' - R\omega^2 = 9.78 \text{m/s}^2$

At poles,  $\phi = 0$ ,  $g' = g = 9.83 \text{m/s}^2$

At depth  $d$  from the surface of earth



$$g' = g \left(1 - \frac{d}{R}\right)$$

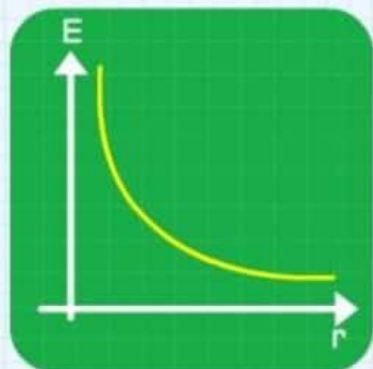
$g' = 0$  if  $d = R$  i.e., at centre of earth

- At equator, effect of rotation of earth is maximum and value of  $g$  is minimum.
- At poles, effect of rotation of earth is zero and value of  $g$  is maximum.



Gravitation field strength at a point in gravitational field is defined as:

$$\vec{E} = \frac{\vec{F}}{m} = \text{Gravitational force per unit mass.}$$



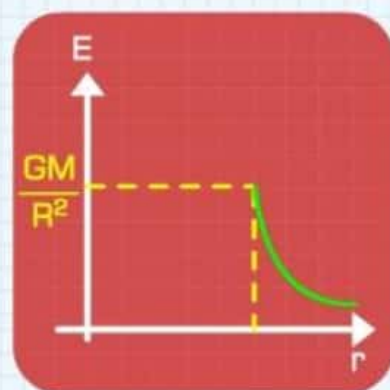
## Due to a point mass

$$E = \frac{GM}{r^2} \text{ (towards the mass)}$$

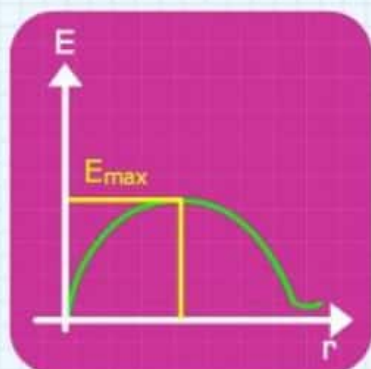
$$\text{or } E \propto \frac{1}{r^2}$$

## Due to spherical shell

- Inside points,  $E_i = 0$
- Just outside the surface,  $E = \frac{GM}{R^2}$ ;  $R$  - Radius of Sphere
- Outside Point,  $E_o = \frac{GM}{r^2}$ ;  $r$  - Distance of centre from an external point
- On the surface  $E$ - $r$  graph is discontinuous.



## On the axis of a ring



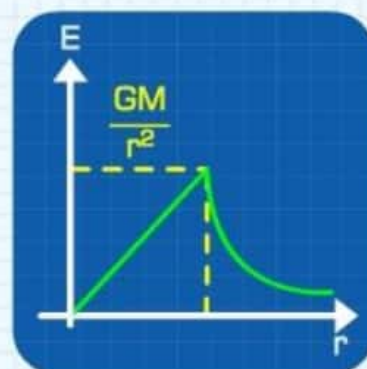
$$E_{ix} = \frac{GMx}{(R^2+x^2)^{3/2}} ; \text{ At } x = 0, E = 0 \text{ i.e., at centre}$$

$$\text{If } x \gg R, E = \frac{GM}{x^2} \text{ i.e., ring behaves as a point mass}$$

$$\text{At } x \rightarrow \infty, E \rightarrow 0 ; E_{\max} = \frac{2GM}{3\sqrt{3}R^2} \text{ at } x = \frac{R}{\sqrt{2}}$$

## Due to a solid sphere

- Inside points  $E_i = \frac{GM}{R^3} r$
- At  $r = 0, E = 0$  i.e. at centre
- At  $r = R, E = \frac{GM}{R^2}$  i.e., on surface
- Outside points  $E_o = \frac{GM}{R^2}$  or  $E_o \propto \frac{1}{r^2}$
- At  $r \rightarrow \infty, E \rightarrow 0$



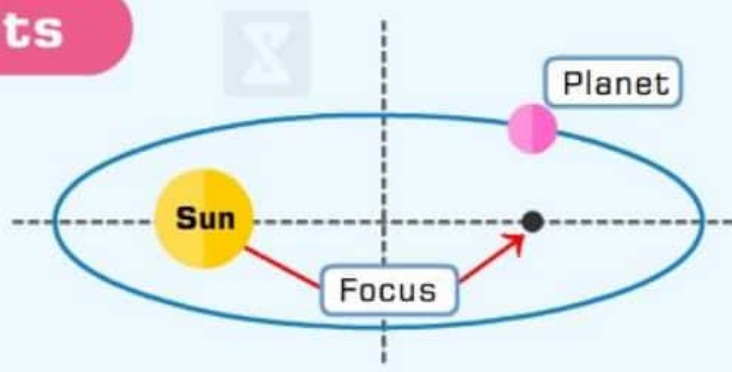


# Kepler's law of Planetary Motion

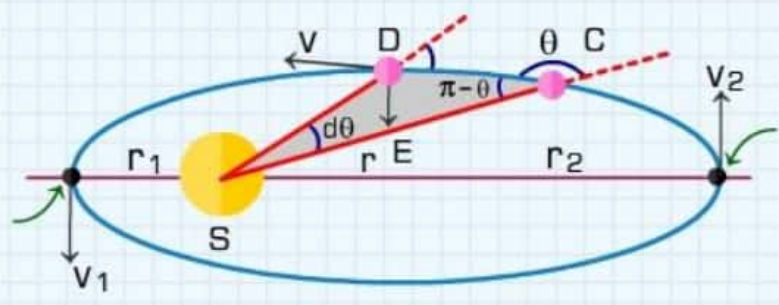


## 1<sup>st</sup> Law The Law of Orbits

All the planets move around the sun in elliptical orbits with sun at one of the focus, not at centre of orbit.



## The Law of Areas 2<sup>nd</sup> Law



The line joining the sun and planet sweeps out equal areas in equal time.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{Constant}$$

## 3<sup>rd</sup> Law The Law of Periods

The time period of revolution of a planet in its orbit around the sun is directly proportionally to the cube of semi - major axis of the elliptical path around the sun.

$$T^2 \propto a^3$$

